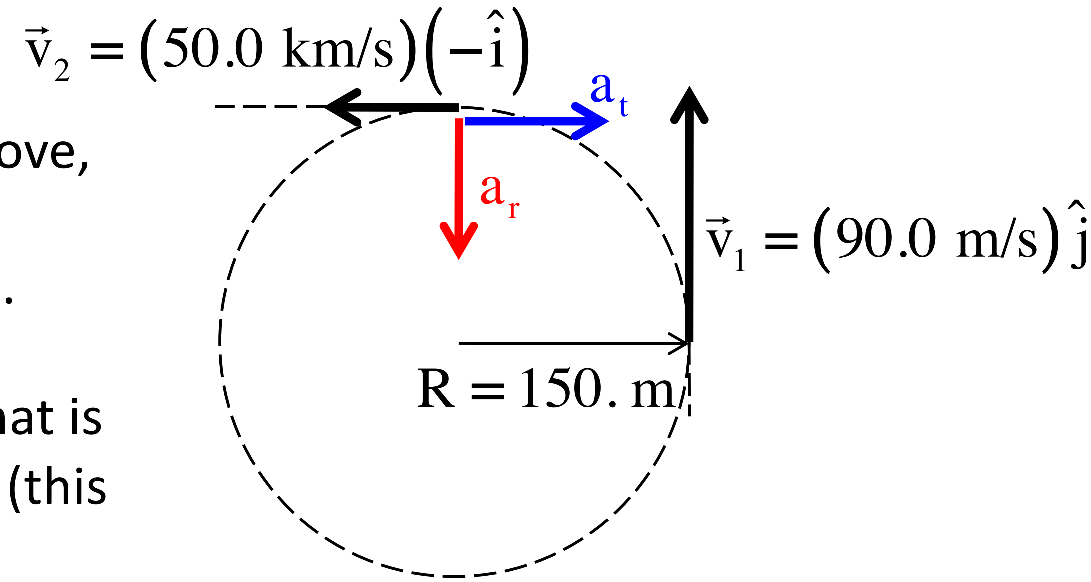
Problem 4.31



The slowing train, as viewed from above, is shown to the right, complete with acceleration vectors at the end-point.

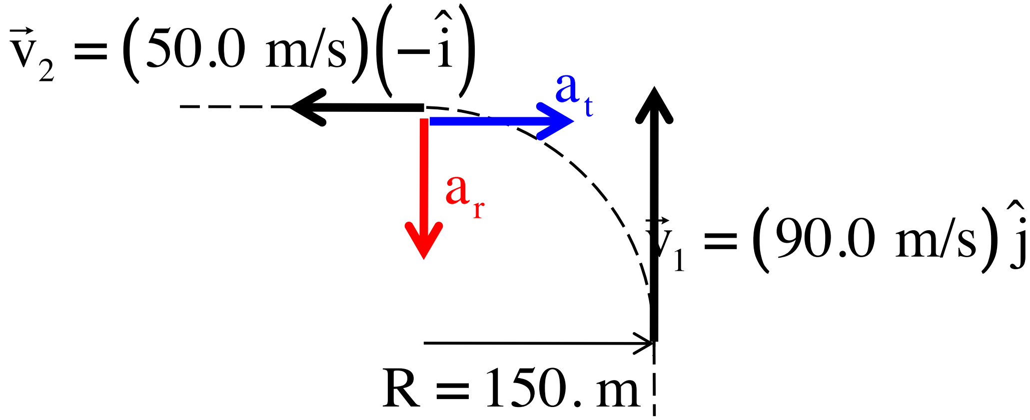
The magnitude of the acceleration that is *changing the direction of the motion* (this is perpendicular to the motion in the centripetal or radial direction) is equal to:

$$a_r = \frac{v^2}{R}$$

$$= \frac{\left[(50.0\ \text{km/hr})\left(\frac{(1.00\text{x}10^3\ \text{m})}{(1.00\ \text{km})}\right)\left(\frac{(1.00\ \text{hr})}{(3.60\text{x}10^3\ \text{s})}\right)\right]^2}{(150.\ \text{m})}$$

$$= 1.29\ \text{m/s}^2$$

The magnitude of the acceleration that is physically *slowing the motion* (this is along the line of the motion in the tangential direction) is equal to:



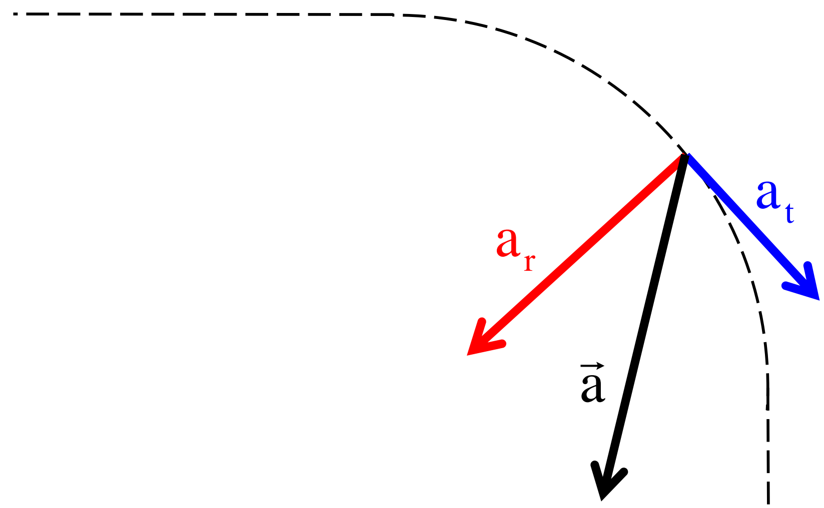
$$a_t = \frac{\Delta|\vec{v}|}{\Delta t}$$

$$= \frac{\left(|\vec{v}_2| - |\vec{v}_1|\right)}{\Delta t}$$

$$= \frac{\left[(50.0 \text{ km/hr})\left(\frac{(1.00\text{x}10^3 \text{ m})}{(1.00 \text{ km})}\right)\left(\frac{(1.00 \text{ hr})}{(3.60\text{x}10^3 \text{ s})}\right)\right] - \left[(90.0 \text{ km/hr})\left(\frac{(1.00\text{x}10^3 \text{ m})}{(1.00 \text{ km})}\right)\left(\frac{(1.00 \text{ hr})}{(3.60\text{x}10^3 \text{ s})}\right)\right]}{(15.0 \text{ s})}$$

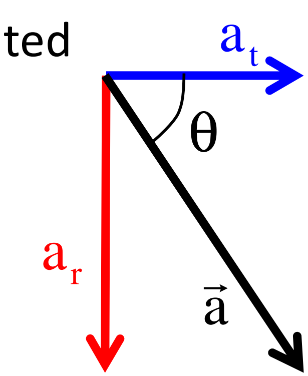
$$= -.741 \text{ m/s}^2$$

Putting everything together yields:



$$\vec{a} = -\left(a_r\right)\hat{r} - \left(a_t\right)\hat{\theta}$$
$$= -\left(1.29 \text{ m}/\text{s}^2\right)\hat{r} - \left(.741 \text{ m}/\text{s}^2\right)\hat{\theta}$$

Noting that the final acceleration vector is in the fourth quadrant if we rotate the axes and measure the angle relative to the tangent (see sketch), our polar calculation will not require the addition of 180^o and we can write:



$$\vec{a} = \left(1.29 \text{ m/s}^2\right)\hat{r} - \left(.741 \text{ m/s}^2\right)\hat{\theta} = \left(\left(1.29 \text{ m/s}^2\right)^2 + \left(-.741 \text{ m/s}^2\right)^2\right)^{1/2} \measuredangle \tan^{-1}\left(\frac{1.29}{-.741}\right)$$

$$\Rightarrow \quad \vec{a} = \left(1.49 \text{ m/s}^2\right)\measuredangle - 60^o$$